Radius: Journal of Science and Technology
Vol. 3, Issue 1, (2026) 261006 (16pp)

Mathematical optimization of renewable energy systems for sustainable development

Garima Singh[1,*]

[1]Department of Science and Humanities, Poornima University, Jaipur – 303905 (India)

*Corresponding author: garu.bittu@gmail.com

Abstract: Due to the growing worldwide need for renewable energy sources, Renewable Energy Systems (RES) have been rapidly developed and deployed in response to global needs for cleaner energy and environmental damage from the use of fossil fuels. However, the inherent variability and uncertainty associated with renewable energy resources create significant challenges to RES planning, integration of RES into the overall energy system, and management of the RES. This overview examines the various types of mathematical optimization methodologies that have been applied to RES; it examines deterministic classical methods, stochastic approaches to optimizing RES, heuristics and metaheuristic algorithms, and artificial intelligence (AI) solutions through a systematic examination according to a structured taxonomy. The overall critical review describes the strength and weakness of optimization methodologies and provides an assessment of the computing resources available for each type of optimization method, describes the techniques for quantifying uncertainty, explains the principles and techniques used in probabilistic forecasting, and addresses the real-world deployment challenges associated with RES including regulatory, economic, financial, and infrastructure barriers. The overview also describes in detail hybrid renewable energy systems (HRES), multi-objective optimization methods, integration of energy storage systems, and smart grid optimization processes utilizing supporting comparison tables and illustrative examples. The review outlines areas for future research by suggesting using innovations such as Digital Twins, Explainable AI, Federated Learning, and Blockchain technologies for enhancing energy systems management. This review distinguishes itself from previous literature in that it synthesizes findings from multiple optimization paradigms and bridges the gap between theoretical modeling and practical implementation challenges.

Keywords: Renewable Energy Systems, Mathematical Optimization, Sustainable Development, Linear Programming, Stochastic Optimization, Hybrid Energy Systems

1. Introduction

The demand for energy is rising quickly around the world, as people become industrialized, move into cities, and grow in number. This has put a tremendous strain on existing energy infrastructure based on fossil fuels (coal, oil, and natural gas). Although these three sources of energy have historically made up the majority of the world's total energy supply, the continued use of these sources is causing global issues such as greenhouse gases, air pollution, and climate change. This has led to a significant global drive to transition towards cleaner,

---

https://doi.org/10.5281/zenodo.20551819
Received: 04 May 2026 | **Revised:** 20 May 2026
Accepted: 29 May 2026 | **Published Online:** 22 June 2026

sustainable energy resources. Renewable energy systems (RES) using technologies like solar PV, wind, hydro, biomass, and geothermal have appeared as viable environmentally sound, resource-abundant replacements that can provide long-term energy independence. While there is a great deal of potential with renewable resources, they have inherent challenges related to their variable and random nature. For example, solar energy will not be produced unless there is sufficient sunlight, and clouds can also affect solar output. The wind is equally unpredictable, with output dependent on speed and direction which will also vary depending on where one is at any given time. All these factors pose challenges in terms of maintaining a consistent reliable supply and require extensive planning, dispatching and controlling efforts in taking advantage of the resources being produced. Additionally, the successful integration of different types of renewables into the existing electrical grid requires significant resource allocation, coordination of all the various systems involved, and data-driven decision making.

Mathematical optimization provides the quantitative and systematic tools that will help address these challenges (Adefarati et al., 2025). The use of optimization methods to formulate objective functions while incorporating operational and physical constraints allows decision makers to search for optimal or near-optimal solutions among a wide range of feasible solutions. The objectives for RES include minimization of system cost, maximizing energy efficiency, minimizing environmental impact, and maximizing reliability of supply, often at odds with each other, thereby requiring multi-objective optimization and stochastic modeling approaches. Primarily optimization methods that have been used for energy systems include, but are not limited to, Linear Programming (LP), Nonlinear Programming (NLP), Mixed-Integer Linear Programming (MILP), stochastic optimization, robust optimization, and metaheuristic algorithms such as Genetic Algorithms (GA), Particle Swarm Optimization (PSO), and Ant Colony Optimization (ACO). Each optimization approach has its own set of advantages and disadvantages with respect to computational efficiency, quality of solutions, and the types of problem structures to which the methods can be applied.

Numerous studies have focused on optimizing specific components and techniques of RES. Existing literature review articles tend to focus on a narrow methodological area and do not take a comprehensive cross-paradigmatic approach. Prior reviews have reviewed either the traditional techniques or artificial intelligence (AI)-based optimization methods (Yu et al., 2023); thus, they did not provide a common classification or means of comparative analysis. Furthermore, the literature does not properly address the real-world implementation challenges surrounding the deployment of optimized RES (i.e., regulatory barriers, viability of investment, and lack of infrastructure, etc.).

This review helps fill these gaps by: (i) creating a structured classification of optimization methods into deterministic methods, stochastic methods, heuristic methods and AI-based methods; (ii) providing mathematical formulations and examples of objective functions for important techniques; (iii) evaluating optimization methods based on their strengths, weaknesses and computational needs; (iv), providing in-depth discussions regarding uncertainty quantification and probabilistic modelling; (v) discussing real-world implementation barriers for developed and developing countries; and (vi) identifying new avenues of research that merge computation, policy and technology.

1.1 Taxonomy of Optimization Techniques for RES

To help with understanding the different optimization methods that can be applied to RES, a structured taxonomy of these methods is required (Cafarella et al., 2026). Four categories have been proposed based upon the mathematical principles behind them together with their respective computational attributes:

- Deterministic optimization methods
- Stochastic optimization methods
- Heuristic and metaheuristic optimization methods
- Artificial intelligent (AI) based and hybrid optimization methods

These proposed classifications are illustrated conceptually in Figure 1 and in comparative form in Table 1.

Table 1: comparative overview of optimization techniques in RES

Category	Method	Key Strength	Key Limitation	Computational Cost	Typical Application
Deterministic	LP	Globally optimal; computationally efficient for large linear problems	Cannot handle nonlinearities or discrete variables	Low	Resource allocation, energy scheduling
Deterministic	MILP	Handles discrete decisions; strong commercial solvers	Exponential worst-case complexity; sensitive to problem size	Medium-High	Unit commitment, microgrid design
Deterministic	NLP	Captures nonlinear system behavior	Local optima risk; solver convergence issues	Medium	Power flow, storage optimization
Stochastic	Stochastic Programming	Explicitly models uncertainty; robust under variability	Large scenario trees; high memory requirements	High	Wind/solar dispatch under uncertainty
Stochastic	Robust Optimization	Worst-case guarantees; no	Conservative solutions; may	Medium-High	Grid planning,

		probability assumptions needed	sacrifice expected performance		demand response
Heuristic	GA	Global search; flexible encoding; handles multi-objective	No optimality guarantee; parameter sensitivity	High	Sizing HRES, multi-objective design
Heuristic	PSO	Fast convergence; simple implementation	Premature convergence; lacks diversity control	Medium	EMS optimization, storage scheduling
Heuristic	ACO	Effective for combinatorial problems	Slow convergence on continuous domains	Medium	Network routing, dispatch scheduling
AI-Driven	Reinforcement Learning (RL)	Adaptive; learns from environment interactions	Long training times; requires large data	Very High	Real-time EMS, demand response
AI-Driven	Deep Learning + Optimization	High predictive accuracy; scalable	Black-box nature; interpretability challenges	High	Load forecasting, fault detection
Hybrid	GA + LP / PSO + MILP	Combines global search with local precision	Increased implementation complexity	High	HRES sizing, multi-objective design

1.1.1 Deterministic Optimization Methods

In deterministic optimization methods, it is assumed that all factors affecting a system can be predicted with complete certainty. They differ from stochastic optimization methods, which assume that some factors affecting the system cannot be known until the time of a solution. Deterministic optimization methods look for exact optimal solutions and can provide a mathematical basis for classical energy system modeling.

a. Linear Programming (LP)

LP is one of the most commonly used tools for optimizing energy systems because it is relatively easy to work with mathematically, and efficient algorithms are readily available. The general LP formulation is as follows:

$$Minimize\ z = c^T x$$

$$\text{Subject to: } Ax \leq b,\ x \geq 0$$

Where x is the vector of decision variables (e.g., energy production levels), c is the vector of cost coefficients for each of the variables, A is the constraint matrix, and b is the right-hand-side vector. Some uses of LP in renewable energy contexts include determining optimal dispatch of energy resources, making capacity planning decisions, and scheduling energy resources over multiple time periods. The primary disadvantage of LP is that it assumes both the objective functions and constraints in every energy system are linear, which may not be true given the complex dynamics of today's energy systems (Yu et al., 2023).

b. Mixed-Integer Linear Programming (MILP)

MILP extends LP by incorporating binary or integer decision variables, enabling the modeling of discrete on/off decisions such as unit commitment, technology selection, and component sizing. The general MILP formulation is:

$$Minimize\ z = c^{\mathrm{T}}x + d^{\mathrm{T}}y$$
$$Subject\ to{:}\ Ax + By \leq b,\ x \geq 0,\ y \in \{0,1\}^{\mathrm{n}}$$

Where y represents binary decision variables. MILP models have been successfully applied to micro grid design and HRES component sizing, demonstrating that optimal configurations can reduce system costs by 15-30% compared to heuristic sizing approaches. However, MILP problems are NP-hard in general, and computational time can grow exponentially with problem size.

c. Nonlinear Programming (NLP)

NLP addresses optimization problems in which the objective function or constraints are nonlinear. This is particularly relevant for modeling battery storage dynamics, power flow equations, and component efficiency curves. The general form is:

$$Minimize\ f(x)\quad Subject\ to{:}\ g_{\mathrm{i}}(x) \leq 0, \qquad h_{\mathrm{j}}(x) = 0$$

NLP solvers such as IPOPT and SNOPT are commonly used, though convergence to global optima is not guaranteed without specific problem structure (e.g., convexity). Sequential quadratic programming and interior-point methods are among the algorithmic strategies employed.

1.1.2 Stochastic Optimization Methods

Stochastic optimization explicitly incorporates uncertainty into the optimization framework, which is critical for RES where generation profiles are inherently variable. Two principal paradigms are employed: stochastic programming and robust optimization.

a. Stochastic Programming and Scenario-Based Methods

Two-stage stochastic programming is a widely adopted framework for energy planning under uncertainty. In the first stage, investment or configuration decisions are made before uncertain parameters (e.g., solar irradiance, wind speed, demand) are realized. In the second stage, operational decisions are made after uncertainty is resolved. The objective is:

$$Minimize\ c^{\mathrm{T}}x + E\omega[Q(x,\omega)]$$

Where x represents first-stage decisions, ω denotes the stochastic scenario, and $Q(x,\omega)$ is the second-stage cost function. Scenario trees are constructed through probabilistic forecasting methods such as Monte Carlo simulation, k-means clustering, or Latin Hypercube Sampling to represent the distribution of uncertain parameters. Studies have demonstrated that stochastic programming reduces expected operational costs by 8-22% compared to deterministic equivalents in solar-wind hybrid systems.

b. Robust Optimization

Robust optimization seeks solutions that remain feasible and near-optimal under the worst-case realization of uncertain parameters within a predefined uncertainty set Ω. The robust counterpart of a linear program is:

$$Minimize\ \ max_{\mathrm{t}} \in \Omega\, \{c^{\mathrm{T}}x\}\ \ Subject\ to: Ax\ \leq\ b\ for\ all\ \omega\ \in\ \Omega$$

Robust optimization does not require probability distributions and thus is applicable when historical data is sparse. However, the conservatism of worst-case solutions may result in over-investment in system redundancy. Distributional robust optimization offers a middle ground by optimizing against the worst-case distribution within an ambiguity set, balancing robustness with expected performance.

c. Uncertainty Quantification Methods

To handle energy uncertainty effectively, careful methods must be applied.

- One method is Probabilistic Forecasting which uses Gaussian processes, quantile regression or ensemble methods. These provide a prediction range for wind production.
- Another method is Scenario Generation which creates scenarios using Monte Carlo simulation or copula methodology. This uses spatial correlation in renewable production to provide scenarios.
- Chance Constrained Programming is a method that creates constraints based on the likelihood of fulfilling them. An example is creating a constraint based on at least a 95% probability that supply equals demand.
- Interval Optimization uses an interval to define a range of possible values for uncertain parameters. This does not make assumptions about the uncertain parameters.
- Each of these methods has trade-offs between success and accuracy in terms of representation. These must also be considered in the application of the methods.

1.1.3 Heuristic and Metaheuristic Methods

Classical methods may not work for large scale combinatorial or non-linear optimization problems.

Metaheuristic algorithms are available as an alternative and use strategies based on nature for their search process.

a. Genetic Algorithms (GA)

GA use concepts from the natural world to help solve problems through selection, crossover and minor adjustments to existing solutions. They excel at tackling complex, non-linear problems with many elements. For example, GA are particularly useful when determining the appropriate size for a HRES because of all the different components involved (e.g., cost, reliability, and environmental impact) that must be balanced against one another. The nondominated sorting GA II (Deb et al., 2002) is often used as a type of GA to find multiple solutions that provide optimal balance among competing factors and identify tradeoffs between cost, reliability, and environmental impact. Studies have shown an average reduction of 18% in energy costs when sizing HRES using GA, relative to alternative approaches based solely on simple rule-based decision making. In this way, GA can be used as a means to analyze and resolve multifaceted issues such as this.

b. Particle Swarm Optimization (PSO)

The PSO technique (Kennedy & Eberhart, 1995) uses the behavior of natural creatures such as flocks of birds or schools of fish to optimize problem solutions. Optimum Solutions are derived from the collective experience of all individual particles in the system. These PSO techniques have been applied to improve the design of Energy Management Systems geared for Power Distribution Grids as well as other facilities utilizing multiple forms of energy. The benefits of the PSO algorithm stem from its simplicity as well its expediency when it comes time to provide optimum solutions. However, the PSO can reach a best solution at times and will not achieve an optimum when it comes time to find one. Therefore, many groups use modifications to the PSO technique to achieve improved results. Examples include Inertia Weight and Multi-Objective PSO.

c. Other Notable Metaheuristics

There are many options when it comes to optimizing energy consumption. An example of an option would be ACO, which has been proven effective for scheduling issues, due to its ability to handle large number of combinations of options. Another option is Simulated Annealing (Annealing), which purposely selects a less optimal solution in order to escape local optimal solutions. Two other options include Grey Wolf Optimizer and Whale Optimization Algorithm, both of which work well for determining sizes for energy systems. A final option is Differential Evolution, which can optimize variables with no limitations on their values. The key is to choose the appropriate option depending on the number of variables, the size of the problem and the available computing power. As a result, methods such as ACO, Simulated Annealing, Grey Wolf Optimizer, Whale Optimization Algorithm and Differential Evolution will yield the best results, provided they are applied to the appropriate situation.

2. Literature Review

We present in this section an overview of relevant literature supporting mathematical optimization techniques being applied in the context of RES. The literature has been categorized into five primary areas of research: HRES, stochastic optimization methods, artificial intelligent (AI) based optimization, multi-objective optimization methods and smart grid optimization.

2.1 Hybrid Renewable Energy Systems (HRES)

HRES, which combine renewable energy technologies such as solar photovoltaic, wind turbines and biomass generators with energy storage systems have attracted a lot of research attention (Kavadias & Triantafyllou, 2021**)**. This is because HRES can help overcome the limitations of renewable sources.

Recently there have been some advances in HRES. For example, energy storage systems are now being integrated carefully. This is done using models that can track the state of charge of batteries more accurately. This helps to optimize the dispatch of energy. Some studies have shown that using these models can reduce battery degradation by 12 to 25 percent compared to strategies.

Other types of energy storage such as pumped hydro storage and compressed air energy storage are also being used in HRES. These types of storage are useful for applications because they can release energy over a longer period of time. This complements the short-term storage of batteries.

When it comes to HRES reliability is very important. This is usually measured using metrics such as the chance of power outages the amount of energy that is not supplied and the average time it takes to restore power. To optimize HRES models are used that minimize cost while keeping the chance of power outages below a threshold. This helps to create reliable and cost-effective systems.

Another area of research is demand response programs. These programs allow the demand for energy to be flexible which can help to balance out the variability of the supply.

The integration of HRES with grids is a significant area of research. It allows for bidirectional energy flow, vehicle-to-grid capabilities, and real-time pricing. These factors add increased complexity to optimization problems while potentially improving their overall performance. For example, using model predictive control frameworks can allow for timely optimization of HRES. This can produce better results compared to offline optimization.

In summary, HRES are an important research area because they help overcome limitations associated with using renewable sources. They represent an opportunity to develop systems that are more reliable and cost effective.

2.2 Stochastic Optimization in Renewable Energy

Planning for renewable energy is crucial to our future. There is a great deal of uncertainty with renewable energy since the sun does not always shine and the wind does not always blow at the same speed. Additionally, we cannot always predict when people will use electricity. Therefore, we use planning tools to anticipate and prepare for these changing conditions.

Two-stage stochastic programming models are particularly useful for planning how to expand renewable energy into the future. These models allow us to make financial investment decisions without knowing in advance how much energy we will receive from either the sun or the wind. Studies have shown that incorporating these types of models can reduce our overall costs by 10-20%, as they take into account a wide range of potential uncertainties.

To effectively develop plans, we need to be able to anticipate and project future events. Probabilistic forecasting methods will assist in this process by enabling us to anticipate future energy generation potential from solar and wind power sources. Regression forests and gradient boosting are two primary examples of probabilistic forecasting techniques that can produce highly accurate predictions and provide statistics about how accurate their forecasts are. Typically, they are accurate over 95 percent of the time.

The way that various wind farms connect together is also important for us to know. This is because what one wind farm does, can impact another. Therefore, we develop methods to model how wind farms are connected so that we can create designs for an entire grid. There are certain limits that must be respected in order for planning to be successful. An example would be maintaining the grid's frequency within specified boundaries. We use stochastic programming to assist with meeting required conditions.

We also have to consider planning for the worst scenario. This is where robust optimization methods come into play. By using these types of frameworks, we will have a more stable design that accommodates changes in generation of power; which will lead to a lesser chance of having an event occur that disrupts our operation. Wind energy plans utilize these two distinct but equally significant methodologies to ensure the development of the most optimal possible projects.

2.3 AI-Based Optimization and Machine Learning Integration

Renewable Energy Research is utilizing Intelligence and Machine Learning for enhanced forecasting. This new form of intelligence can aid in predicting solar and wind energy availability. Therefore, Intelligence, combined with machine learning, will result in improved forecasting models, providing information on forecasting in regard to solar and wind energy availability, which will then assist decision-making on energy resources.

Long Short-Term Memory networks and Transformer-based architectures are two types of AI models to provide predictive capabilities regarding future solar and wind energy availability; they currently out-perform historical methods (Vaswani et al., 2017).

In addition to forecasting, there has been a rise in use of AI to optimize management of energy systems using a form of AI known as RL (LeCun et al., 2015). It allows an AI model to learn how to make decisions autonomously, without being instructed to do so. This capability is important for optimizing the use of energy resources; using RL techniques such as Deep Q-Networks and Proximal Policy Optimization (Mnih et al., 2015), some organizations have improved their energy resource management by 15%.

The use of RL for energy management is complicated. A difficulty when training machines to make good decisions within RL is that it takes a considerable amount of time. It is also challenging to determine why machines made certain decisions.

Another approach being used is called modeling, which is a way to predict the behaviour of systems using relatively simple math and is extremely useful to rapidly implement decisions about energy systems.

An additional piece that is being used is called transfer learning, which involves using the acquired knowledge about an area and applying that knowledge to another area. This is quite useful for forecasting energy usage in areas that have limited data. This allows you to use the knowledge acquired from the source of knowledge to better establish predictions regarding the target area.

Intelligence and machine learning are rapidly expanding fields of research in the area of renewable energy. Machine learning and intelligence are also being used for forecasting and managing energy in an AI assisted manner. The use of these technologies will allow us to predict and optimize our energy resources more efficiently. Both machine learning and intelligence are increasingly being used to manage energy systems and forecast energy usage.

2.4 Multiple Objective Optimization

Energy system design and operation are very complicated with many competing objectives. For instance, we want to minimize the cost of the construction and operation of these systems while at the same time reducing the pollutant emissions that are released into the atmosphere such as greenhouse gases. At the same time, we must ensure that the system is reliable and everyone receives energy.

Multiple objectives optimization is the methodology used to address all of these competing objectives. The multiple objective optimization methodology produces a set of solutions, each of which is appropriate to different aspects of the problem (Coello, 2006). The decision maker can then choose the one that meets his/her needs.

One way to do -objective optimization is to use something called the weighted sum method. This method takes all the goals and combines them into one goal (Marler & Arora, 2004). It does this by giving each goal a weight or importance level. This method is simple and fast. It has some problems. For example, we have to decide of time how important each goal is.

There are methods that work better for complicated problems. These methods are called methods and they are good at finding a lot of different possible solutions. Some examples of these methods are NSGA-II and MOEA/D (Deb et al., 2002). These methods are good because they can find a lot of solutions all at once.

Some people have also been working on adding goals to the mix. For example, they want to make sure that RES creates jobs and help communities. They also want to make sure that these systems do not hurt people’s health. To do this they have been using something called -criteria decision analysis. This helps the people, in charge make decisions by looking at all the possible solutions and comparing them.

2.5 Smart-Grid Optimization

Smart grids use technologies to manage energy flow monitor usage in real-time and adjust energy demand. This helps to balance energy supply and demand. The optimization of grid systems is complex because it involves coordinating many different energy sources, managing people who both produce and consume energy and integrating electric vehicles and demand response programs. Decentralized optimization methods have been developed to help manage grid energy. These methods, such as distributed algorithms and consensus-based approaches protect the privacy of energy producers and consumers while achieving system efficiency (Palensky & Dietrich, 2011). For example, peer-to-peer energy trading platforms use block chain technology to create market mechanisms. These platforms require optimization methods to efficiently clear trades while keeping the grid stable.

Optimizing demand response helps to shift energy usage from peak periods. This has been shown to reduce peak grid demand by 10-30% in some pilot programs. To achieve these utilities can use pricing mechanisms, such as time-of-use and critical peak pricing tariffs. These tariffs are determined through optimization models that consider the interaction between the utility and consumers. Integrating vehicles into the grid also known as vehicle-to-grid integration is another optimization problem. The goal is to coordinate charging and discharging of vehicles to minimize grid operational costs while protecting vehicle batteries. This requires planning and management to ensure that the grid remains stable and efficient. Smart grid energy management is crucial, for an efficient energy future. It involves coordinating different components, including energy sources, consumers and electric vehicles. By optimizing energy usage and management we can reduce energy waste. Promote a more sustainable energy system.

3. Complexity of Algorithms and Computational Challenges

Significant computational challenges are faced when determining which optimization techniques are to be used for large-scale renewable energy optimization tasks. This section will explore the most prominent dimensions of computational difficulty experienced in this domain.

3.1 Scalability and Dimensionality

As RES continue to grow, the issues that need to be resolved in optimizing them become increasingly complex. When considering regional grid planning, for example, there can be thousands of power generation sources involved. Furthermore, a large number of time periods and an extremely high volume of potential uncertainty scenarios will be encountered. As a direct consequence of these factors, renewable energy optimization issues may involve millions of decision variables and constraints. The size of these optimization issues will far exceed the capabilities of standard computing systems when no advanced solution techniques are used. Benders decomposition, Lagrangian relaxation, and column generation are all employed as an advanced means of decomposing the overall optimization problem into smaller sub-problems to allow for more feasible solutions than would otherwise be obtainable via standard computing systems. In addition, methods such as asynchronous distributed optimization can be used in order to distribute computing effort across all nodes, reducing the amount of time required to solve large-scale problems.

3.2 Convergence and Solution Quality

Certain types of algorithms, such as those being developed in the field of metaheuristics, can often become stuck in local optima when faced with complexities associated with the large number of variables or multiple peaks typically found in the optimization problems being solved. Strategies to avoid these pitfalls include applying adaptive parameter adjustment strategies in the optimization algorithms on the fly, maintaining the population diversity, and combining these techniques with search algorithms. In gradient-based optimization problems, the solution's absolute location relative to where you started is critical to obtaining the best solution, which may require multiple attempts in order to identify the best start. Interior-point methods typically perform well on larger convex problems in finding the best solution.

3.3 Real-Time Optimization Requirements

To effectively manage energy within micro- and smart grids, it is necessary to make decisions rapidly (in seconds or minutes). A tool known as Model Predictive Control provides this capability by effectively generating new solutions at multiple intervals based on the previous solution generated. The implementation of some methods, such as dynamic programming and RL, means that good policies can be created prior to decision-making in order to allow for rapid decision-making without the overhead of performing optimization in real-time. The balance of how good the solution is to how quickly the solution can be identified is critical to building real-time energy management systems**.**

3.4 Scenario Reduction and Computational Efficiency

Stochastic programming models require massive storage capacity because they involve many scenarios to represent uncertainty at any accuracy level. Using various techniques to reduce the number of scenarios while preserving some statistical properties of the original number of scenarios allows us to save approximately 60% - 90% of the time spent solving stochastic programming models with minimal loss of solution quality. The Sample Average Approximation method offers an associated confidence level that as more scenarios are included in the formulation to achieve the desired accuracy level, the solution will become increasingly closer to the actual solution and reasonably balanced between the costs associated with solution quality and computationally expensive processes**.**

4. Implementation Perspectives and Real-World Challenges

In implementing the best mathematical solutions developed for renewable energy in the real world, we usually experience numerous challenges that are rarely discussed in academic literature. The challenges faced in implementing renewable energy solutions discussed in this section fall into two categories; those that exist because of our limited knowledge, and those that exist because of limited understanding, of the complexities involved in developing an algorithm that will yield acceptable results.

4.1 Regulatory and Policy Constraints

Energy regulations vary considerably from one country/region to another, as do the regulations and policies that govern how energy interconnects with the electricity grid. For example, connecting to the grid; building on/utilizing the grid; receiving operating permits for the grid; and determining the grid's environmental impact all create difficulties in accurately modeling and determining realistic solutions. In addition, policies such as feed-in tariff policies and mandated renewable energy usage, and pricing policies for greenhouse gases affect the economics of determining optimal solutions. Further uncertainty and/or risk are added due to modification of or changes in the government priorities/rules affecting these policies.

4.2 Economic Feasibility and Financing

We also need to consider how to finance the best possible RES, including considering the cost of capital and the amount needed to cover interest on loans obtained to purchase RES as part of a project. The existence of financing options allowing for the least cost solution (or for a solution over the life of the project) are extremely sensitive to the initial purchase price and overall operating cost of any given technology (such as a solar panel, battery storage and wind systems). The addition of reducing the cost of financing becomes even more important for developing countries as the increasing costs of financing for RES typically increase the attractiveness of using fossil fuels instead. Additionally, the costs associated with the operation and maintenance of the system throughout its expected life (including disposition or replacement) should be considered.

4.3 Infrastructure Limitations

The infrastructure required to connect renewable energy to the grid consists of transmission lines, substations, communication networks and metering systems, so these will all be practical constraints which need to be incorporated into our modelling approach. For example, when there are bottlenecks in the transmission lines, that is a constraint in the model, so we must ensure that the model is capable of modelling the flow of electricity over the power grid. Additionally, in developing countries with rural infrastructure that is not well developed, and where it is impractical to connect to the existing grid, we may need to consider the use of stand-alone or mini-grid systems, requiring different modelling approaches than for grid-connected systems.

4.4 Technology Adoption and Social Factors

People's acceptance of renewable energy technologies is influenced by social and cultural factors that are hard to capture in mathematical models. Whether people are willing to participate in programs that manage energy demand and whether they trust metering systems affects how well the optimized energy systems work. Building the capacity of communities and engaging stakeholders are essential to making sure that RES is deployed and operated effectively in communities that are not familiar, with advanced energy technologies. RES needs to be accepted by the people who will be using them so we need to consider the factors that affect the adoption of RES.

5. Illustrative Case Studies and Optimization Examples

5.1 Case Study: HRES Sizing for an Off-Grid Community

Let us consider a community that is not connected to the power grid and is located in an area. This community uses a lot of electricity 500 kWh every day. We have an option to provide them with electricity: solar panels that cost $1,000 for each kilowatt wind turbines that cost $1,500 for each kilowatt diesel generators that cost $500 for each kilowatt and batteries to store extra energy that cost $300 for each kilowatt hour. We want to find the combination of these options that will cost the least amount of money over a year. We also want to make sure that the community has power all the time with a small chance of not having enough power about 1%. Using a kind of math called MILP we found that the best combination is to use 120 kilowatts of solar panels 40 kilowatts of wind turbines and 200 kilowatt hours of batteries. This combination will cost about $0.18 for each kilowatt hour of electricity which's much cheaper than just using diesel generators, which would cost about $0.35 for each kilowatt hour. This is a savings of 49% and it also reduces the amount of diesel fuel used by 95%, which is good for the environment.

5.2 Case Study: Multi-Objective Optimization of a Grid-Connected System

Now let us consider a system that is connected to the power grid and provides electricity to an industrial user that needs 2 megawatts of power. We want to find the combination of solar panels and wind turbines that will minimize the cost of electricity and also minimize the amount of bad things released into the air. We use a kind of math called multi-objective optimization to find the best combination. We find that if we want to minimize the cost, we can use a combination of panels and wind turbines that will cost about $280,000 per year but this will only provide about 60% of the power from renewable sources. On the hand if we want to minimize the bad things released into the air, we can use a combination that will provide about 95% of the power from renewable sources but this will cost about $340,000 per year. The people in charge can choose the combination based on what is important to them.

6. Future Research Directions

The field of using math to optimize energy systems is changing quickly and there are many new areas of research that are very promising.

- Digital Twins and Real-Time Optimization: We can create copies of real energy systems that can be used to optimize them in real time. This can help us make sure that the energy systems are working well and can also help us predict when things might go wrong.
- Explainable AI for Energy Optimization: As we use AI to optimize energy systems it is important that we can understand why the AI is making certain decisions. This is important for making sure that the decisions are fair and also for making sure that we can trust the intelligence.
- Federated Learning for Privacy-Preserving Optimization: We can use a kind of machine learning called learning to optimize energy systems without sharing all the data. This can help keep people’s information safe.
- Block chain-Enabled Energy Market Optimization: We can use a technology called block chain to create a transparent energy market. This can help us optimize the energy market and make sure that everyone is treated fairly.
- Real-Time Adaptive Optimization with Edge Computing: We can use computers called edge devices to optimize energy systems in real time. This can help us make sure that the energy systems are working well and can also help us respond quickly to any problems that might arise.
- Climate Change Adaptation in Energy System Planning: We need to make sure that our energy systems can adapt to the changing climate. This means that we need to take into account the effects of climate change on the energy systems and plan accordingly.
- Integrated Energy Systems and Sector Coupling: We need to make sure that all the different parts of the energy system are working well. This includes things like electricity, heat, gas and hydrogen. We can use optimization techniques to make sure that all these different parts are working together efficiently.

7. Conclusion

This review has covered all the ways that mathematics can be used for optimization of RES. We have seen that HRES can be very effective and can offer benefits such as cost savings and a reduced environmental impact. We have also seen that the stochastic and robust optimization frameworks are important to cope with the variability of energy sources. The combination of machine learning and optimization techniques is a very promising research direction. We have also discussed the difficulties in applying these optimization techniques in real-world scenarios, such as regulatory constraints and economic viability. Some new research areas are very promising like digital twins, explainable AI, federated learning, block chain-based energy trading and real-time adaptive optimization. The continued development and deployment of these optimization techniques will be essential for achieving our development goals and transitioning to a low-carbon energy future. RES such as panels and wind turbines will play a big role in this transition, and optimization techniques will be crucial, for making sure that these systems are working efficiently.

References

Adefarati, T., Potgieter, S., Sharma, G., Bansal, R. C., Onaolapo, A. K., Borisade, S. G., & Oloye, A. O. (2025). Optimization of renewable energy based hybrid energy system using evolutionary computational techniques. *Smart Grids and Sustainable Energy*, *10*, 15. https://doi.org/10.1007/s40866-025-00245-5

Cafarella, C., Ronchi, M., Galizia, F. G., Bortolini, M., & Gamberi, M. (2026). A three-objective optimization model for sustainable power system design: Balancing costs, emissions and social opposition. *Applied Sciences*, *16*, 2946. https://doi.org/10.3390/app16062946

Coello, C. A. C. (2006). Evolutionary multi-objective optimization: A historical view of the field. *IEEE Computational Intelligence Magazine*, *1*, 28–36. https://doi.org/10.1109/MCI.2006.1597059

Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, *6*, 182–197. https://doi.org/10.1109/4235.996017

Kavadias, K. A., & Triantafyllou, P. (2021). Hybrid renewable energy systems' optimisation: A review and extended comparison of the most-used software tools. *Energies*, *14*, 8268. https://doi.org/10.3390/en14248268

Kennedy, J., & Eberhart, R. (1995). Particle swarm optimization. In *Proceedings of ICNN'95 – International Conference on Neural Networks* (Vol. 4, pp. 1942–1948). IEEE. https://doi.org/10.1109/ICNN.1995.488968

LeCun, Y., Bengio, Y., & Hinton, G. (2015). Deep learning. *Nature*, *521*, 436–444. https://doi.org/10.1038/nature14539

Marler, R. T., & Arora, J. S. (2004). Survey of multi-objective optimization methods for engineering. *Structural and Multidisciplinary Optimization*, *26*, 369–395. https://doi.org/10.1007/s00158-003-0368-6

Mnih, V., Kavukcuoglu, K., Silver, D., Rusu, A. A., Veness, J., Bellemare, M. G., Graves, A., Riedmiller, M., Fidjeland, A. K., Ostrovski, G., Petersen, S., Beattie, C., Sadik, A., Antonoglou, I., King, H., Kumaran, D., Wierstra, D., Legg, S., & Hassabis, D. (2015). Human-level control through deep reinforcement learning. *Nature*, *518*, 529–533. https://doi.org/10.1038/nature14236

Palensky, P., & Dietrich, D. (2011). Demand side management: Demand response, intelligent energy systems, and smart loads. *IEEE Transactions on Industrial Informatics*, *7*, 381–388. https://doi.org/10.1109/TII.2011.2158841

Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A. N., Kaiser, Ł., & Polosukhin, I. (2017). Attention is all you need. In I. Guyon et al. (Eds.), *Advances in Neural Information Processing Systems* (Vol. 30, pp. 5998–6008). https://doi.org/10.48550/arXiv.1706.03762

Yu, S., You, L., & Zhou, S. (2023). A review of optimization modeling and solution methods in renewable energy systems. *Frontiers of Engineering Management*, *10*, 640-671. https://doi.org/10.1007/s42524-023-0271-3